

February 1, 2008

Revisiting Signature of Minimal Gauge Mediation

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We revisit phenomenology of the minimal gauge-mediated model. This model is motivated from the SUSY CP and flavor problems. A specific feature of this model is that $\tan\beta$ is naturally large, since the B term in the Higgs potential is zero at the messenger scale. This leads to significant SUSY contributions to various low-energy observables. We evaluate the anomalous magnetic moment of the muon and the branching ratio of $\overline{B} \rightarrow X_s \gamma$ taking account of recent theoretical and experimental developments. We find that the current experimental data prefer a low messenger scale (~ 100 TeV) and gluino mass around 1 TeV. We also calculate the branching ratios of $\overline{B} \rightarrow X_s l^+ l^-$, $B_s \rightarrow \mu^+ \mu^-$, and $B^- \rightarrow \tau^- \overline{\nu}$, and show that these observables are strongly correlated with each other in this model.

1 Introduction

Low-energy supersymmetry (SUSY) is a very attractive model of physics beyond the standard model (SM). In the minimal supersymmetric standard model (MSSM), however, general SUSY breaking masses of squarks and sleptons induce too large FCNC and/or CP violation effects in low-energy observables. These SUSY FCNC and CP problems should be solved in realistic SUSY breaking models.

Gauge-mediated SUSY breaking [1, 2, 3, 4] is one of the promising mechanisms to describe the SUSY breaking sector in the MSSM. The SUSY breaking is transmitted to the MSSM sector through the gauge interaction, which induces the flavor-blind soft SUSY breaking masses of squarks and sleptons. In the minimal gauge-mediated model (MGM) [5, 6], the trilinear scalar couplings (A terms) and Higgs bilinear coupling (B term) are zero at tree level, and they are induced from radiative corrections of the gaugino masses. In this case, dangerous SUSY CP phases are absent, and the SUSY CP problem is solved.

In Refs. [7, 8], phenomenology of the MGM was studied. One of the specific features in the MGM is that $\tan \beta$ is naturally large, and significant SUSY contributions are expected in various low-energy observables. They considered anomalous magnetic moment of the muon, the $\overline{B} \rightarrow X_s \gamma$ decay, and the $\overline{B} \rightarrow X_s l^+ l^-$ decay. It was shown that the deviations of $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and anomalous magnetic moment of the muon from the SM predictions are strongly correlated in the MGM.

Recently, theoretical calculations in both $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and anomalous magnetic moment of the muon have been improved. Also the experimental data was updated. According to Refs. [9, 10], a theoretical estimation of $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ at next-to-next-to-leading order (NNLO) is lower than the experimental world average at a 1.4σ level. In Refs. [11, 12] the SM prediction of anomalous magnetic moment of the muon has been updated by using the recent $e^+ e^- \rightarrow \pi^+ \pi^-$ data. The new SM prediction is larger than the experimental data at a 3.4σ level. While we can not still conclude that the deviations come from the new physics, there is a room for additional new physics contributions.

In this paper, we revisit phenomenology of the MGM taking account of the recent developments of $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and anomalous magnetic moment of the muon. Also, we evaluate $\mathcal{B}(\overline{B} \rightarrow X_s l^+ l^-)$, $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$, and $\mathcal{B}(B^- \rightarrow \tau^- \overline{\nu})$. The SUSY contributions

to all the observables are enhanced when $\tan\beta$ is large. We show that the deviations from the SM predictions are strongly correlated in the MGM and the recent result of $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ prefers a low messenger scale and gluino mass around 1 TeV. This model will be tested by various low-energy experiments in addition to the SUSY direct search at LHC.

2 Minimal Gauge-Mediated Model

First, let us briefly review the MGM. In the gauge-mediated SUSY breaking, the SUSY breaking is mediated to the MSSM sector through the SM gauge interactions. In the MGM, N pairs of $SU(2)_L$ doublet and of $SU(3)_C$ triplet chiral superfields are introduced as messenger fields. The $SU(2)_L$ doublets and of $SU(3)_C$ triplets are assumed to be in $SU(5)$ $\mathbf{5}$ and $\mathbf{5}^*$ -dimensional multiplets. The messenger fields couple to a singlet chiral field by the following superpotential,

$$W = \lambda_i S \Phi_i \overline{\Phi}_i, \quad (1)$$

where S is a singlet, $\Phi_{2(3)}$ and $\overline{\Phi}_{2(3)}$ are $SU(2)_L$ -doublet ($SU(3)_C$ -triplet) superfields. The scalar component of S develops a vacuum expectation value ($\langle S \rangle$), which gives supersymmetric mass terms to the messenger fields, $M_{M_i} = \lambda_i \langle S \rangle$. Also the non-vanishing F -component of S ($\langle F_S \rangle$) induces the SUSY breaking in the messenger fields.

The gaugino masses in the MSSM are generated by one-loop diagrams of the messenger fields and given by

$$\overline{M}_i = N \frac{\alpha_i}{4\pi} \Lambda g(x_i) \equiv \hat{M}_i g(x_i), \quad (2)$$

where $\Lambda = \langle F_S \rangle / \langle S \rangle$ and $x_i = \Lambda / M_{M_i}$. The loop function $g(x_i)$ is given in Ref [13], but we take $x_i \lesssim 1$ in this paper so that $g(x_i) \simeq 1$. Hereafter, the parameters at the messenger scale M_M are denoted by bar. The sfermion mass terms are generated by two-loop diagrams and given by

$$\overline{m}_\alpha^2 \simeq \frac{1}{N} \left(2C_3 \hat{M}_3^2 + 2C_2 \hat{M}_2^2 + \frac{6}{5} Y^2 \hat{M}_1^2 \right), \quad (3)$$

when $x_i \lesssim 1$. Here, C_i and Y are the quadratic Casimir and the hypercharge of sfermions, respectively. In the gauge-mediated SUSY breaking, the squarks become rather heavy

compared with the sleptons and Higgs bosons since the soft SUSY breaking terms are proportional to the gauge coupling constants. In the MGM the trilinear scalar couplings (A) and the bilinear Higgs coupling (B) are assumed to vanish at the messenger scale,

$$\overline{A} = \overline{B} = 0. \quad (4)$$

Those A and B terms are induced by the gaugino-loop diagrams at the weak scale, and the relative phases between the gaugino masses and the A and B terms vanish, $\arg(M_i A^*) = \arg(M_i B^*) = 0$. Therefore, the dangerous SUSY CP phases are absent, and the SUSY CP problem is solved naturally in the MGM.

With the boundary conditions at the messenger scale, we can evaluate the SUSY breaking parameters at the weak scale by solving the renormalization group equations (RGEs). For the μ parameter we do not specify its origin, and it is determined by imposing the correct electroweak symmetry breaking at the weak scale, as

$$\mu^2 = \lambda_t^2 \Delta_t^2 - m_H^2 - \frac{1}{2} m_Z^2 (1 + \delta_H), \quad (5)$$

where λ_t is the top-quark Yukawa coupling, m_H is the common Higgs boson mass, and Δ_t^2 and δ_H represent radiative corrections defined in Ref. [6].

The B term is generated radiatively at the weak scale. It has two types of contributions: one is a Higgsino-gaugino contribution (B_G) and the other is an effective A -term contribution (B_A). They are destructive to each other, and the magnitude of the B parameter is even smaller than one naively expected. We follow the result in Ref. [6] for the numerical calculation of the B parameter. Once the B parameter is fixed, $\tan \beta$ is determined by the minimization condition of the Higgs potential,

$$\tan \beta = -\frac{m_A^2}{B\mu}, \quad (6)$$

where m_A is the CP-odd Higgs boson mass.

Notice that $\tan \beta$ is not a free parameter in the MGM, but, is predicted from other parameters. Since the B parameter is generated radiatively, $\tan \beta$ becomes naturally large. Also, $\text{sign}(B)$ is opposite to $\text{sign}(M_2)$ in most parameter regions. Then $\text{sign}(M_2 \mu \tan \beta)$ is determined to be positive from Eq. (6). Moreover, in the MGM, all the soft SUSY breaking parameters are determined by only three input parameters, Λ , M_M , and N . These are

specific features of the MGM and very important when we consider phenomenology of the MGM.

3 Low-Energy Observables

3.1 Anomalous Magnetic Moment of the Muon

The current experimental measurement of the anomalous magnetic moment of the muon, $a_\mu = (g - 2)_\mu/2$, was reported by Muon (g-2) Collaboration [14],

$$a_\mu^{\text{exp}} = 11659208.0(6.3) \times 10^{-10}. \quad (7)$$

Recently, the SM prediction of a_μ has been updated based on the new data of $e^+e^- \rightarrow \pi^+\pi^-$. The most recent calculation including new evaluation of the hadronic light-by-light scattering contribution is [11, 12]

$$a_\mu^{\text{SM}} = 11659178.5(6.1) \times 10^{-10}. \quad (8)$$

The discrepancy between the experimental data and the SM prediction is

$$\delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (29.5 \pm 8.8) \times 10^{-10}. \quad (9)$$

The most recent result indicates a 3.4σ deviation, and it suggests new physics contributions.

In the MGM, the contribution to a_μ is approximately given by

$$\delta a_\mu^{\text{MGM}} \simeq \frac{5\alpha_2 + \alpha_Y}{48\pi} \frac{m_\mu^2}{M_{\text{SUSY}}^2} \text{sign}(M_2\mu) \tan\beta. \quad (10)$$

Here we assume that all the SUSY mass parameters to be equal to M_{SUSY} . The chargino-sneutrino loop diagram dominates over other diagrams. The SUSY contribution is enhanced when $\tan\beta$ is large and its sign is determined by $\text{sign}(M_2\mu)$. In the MGM, the sign of $M_2\mu \tan\beta$ is positive, and $\delta a_\mu^{\text{MGM}} > 0$. This agrees with the sign of the current discrepancy of a_μ in Eq. (9).

3.2 $\overline{B} \rightarrow X_s \gamma$ Decay

The experimental world average $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ by the Heavy Flavor Working Group [15] is

$$\mathcal{B}(\overline{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}} = (3.55 \pm 0.26) \times 10^{-4}, \quad (11)$$

with a photon energy cut $E_\gamma > 1.6 \text{ GeV}$. The SM prediction of $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ has been estimated by including NNLO contributions in Ref. [9]. Recently, additional perturbative corrections related with the photon energy cut has been calculated [10]. Combining those results, the most recent theoretical prediction is given by [10]

$$\mathcal{B}(\overline{B} \rightarrow X_s \gamma)_{E_\gamma > 1.6 \text{ GeV}} = (2.98 \pm 0.26) \times 10^{-4}. \quad (12)$$

The new SM prediction becomes lower than the experimental data, and the deviation is 1.4σ . While the deviation is not significant, the recent result opens new room for additional new physics contributions.

The $\overline{B} \rightarrow X_s \gamma$ decay is described by the following effective Hamiltonian [9],

$$H_{\text{eff}}^{b \rightarrow s \gamma} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^8 C_i(\mu_b) O_i(\mu_b), \quad (13)$$

where $\mu_b \simeq m_b$. The SUSY contributions are encoded to the Wilson coefficients, C_i . The most relevant operator for the $\overline{B} \rightarrow X_s \gamma$ decay is

$$O_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu}. \quad (14)$$

The calculation of $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ at NNLO is quite complicated. Here we refer Refs. [9, 10] to the formula.

In the MGM, the dominant SUSY contributions to C_7 come from the charged Higgs boson, the chargino, and the gluino loop diagrams. It is known that the charged Higgs amplitude is always constructive with the SM contribution, while the chargino and the gluino loop amplitudes are either constructive or destructive with the SM one. In the mass insertion approximation, the pure-Higgsino ($C_7^{\tilde{h}^-}$), the Higgsino-wino ($C_7^{\tilde{h}\tilde{W}^-}$), and the gluino contributions ($C_7^{\tilde{g}}$) are written as [6]

$$C_7^{\tilde{h}^-} \simeq \frac{1}{2} r_b A_t \mu \tan \beta \frac{m_t^2}{m_{\tilde{Q}}^4} f_{\text{ch}}(\mu^2/m_{\tilde{Q}}^2), \quad (15)$$

$$C_7^{\tilde{h}\tilde{W}^-} \simeq \frac{(\delta_{LL})_{23}}{V_{tb}V_{ts}^*} r_b \tan \beta \frac{M_2 \mu}{M_2^2 - \mu^2} \left[f_{\text{ch}}(M_2^2/m_{\tilde{Q}}^2) - f_{\text{ch}}(\mu^2/m_{\tilde{Q}}^2) \right], \quad (16)$$

$$C_7^{\tilde{g}} \simeq \frac{8\alpha_s}{9\alpha_2} \frac{m_W^2}{m_{\tilde{Q}}^2} \frac{(\delta_{LL})_{23}}{V_{tb}V_{ts}^*} r_b \tan \beta \frac{M_3 \mu}{m_{\tilde{Q}}^2} f_{\text{gl}}(M_3^2/m_{\tilde{Q}}^2), \quad (17)$$

where $m_{\tilde{Q}}$ is an average squark mass and the loop functions f_{ch} and f_{gl} are defined as

$$f_{\text{ch}}(x) = \frac{13 - 7x}{6(x-1)^3} - \frac{3 + 2x(1-x)}{3(x-1)^4} \log x, \quad (18)$$

$$f_{\text{gl}}(x) = -\frac{1 + 10x + x^2}{2(x-1)^4} + \frac{3x(1+x)}{(x-1)^5} \log x. \quad (19)$$

The non-holomorphic correction to the bottom-quark Yukawa coupling is not negligible in large $\tan \beta$ [16]. The correction is represented as $r_b = 1/(1 + \epsilon_b)$, and ϵ_b in it is

$$\epsilon_b = \frac{2\alpha_s}{3\pi} M_3 \mu \tan \beta f(m_{\tilde{Q}_L}^2, m_{\tilde{D}_R}^2, M_3^2). \quad (20)$$

The loop function f in ϵ_b is defined as

$$f(x, y, z) = \frac{xy \log(x/y) + yz \log(y/z) + zx \log(z/x)}{(x-y)(y-z)(z-x)}. \quad (21)$$

The flavor-violating parameter $(\delta_{LL})_{23}$ is induced by the RGE effect and is written in the leading logarithmic approximation as

$$(\delta_{LL})_{23} \equiv \frac{(m_{\tilde{Q}}^2)_{23}}{m_{\tilde{Q}}^2} \simeq -\frac{\lambda_t^2}{4\pi^2} V_{tb}V_{ts}^* \log\left(\frac{M_M}{M_3}\right). \quad (22)$$

With the boundary condition $\overline{A} = 0$, A_t is induced by the RGE effect and proportional to the gaugino masses. As discussed in the previous section, $\text{sign}(M_2 \mu \tan \beta)$ is predicted to be positive in the MGM. In this case $\text{sign}(A_t \mu \tan \beta)$ is also positive and the sign of the pure-Higgsino amplitude is opposite to that of the charged Higgs contribution. Then the dominant SUSY contributions can cancel with each other. The Higgsino-wino and gluino contributions are also constructive with the pure-Higgsino one.

In order to calculate $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ at NNLO in the MGM, SUSY contributions should be matched into C_i beyond LO at the weak scale. But this is beyond the scope of this paper. For the numerical calculation, we use the LO SUSY corrections to C_i at the weak scale while we have included the SM contributions at NNLO.

3.3 $\overline{B} \rightarrow X_s l^+ l^-$ Decay

$\mathcal{B}(\overline{B} \rightarrow X_s l^+ l^-)$ in a low dilepton invariant mass region, $1 < m_{ll}^2 < 6 \text{ GeV}^2$, was reported by Babar, $(1.8 \pm 0.9) \times 10^{-6}$ [17] and Belle, $(1.5 \pm 0.6) \times 10^{-6}$ [18]. The current world average is given by

$$\mathcal{B}(\overline{B} \rightarrow X_s l^+ l^-)_{1 < m_{ll}^2 < 6 \text{ GeV}^2} = (1.6 \pm 0.51) \times 10^{-6}. \quad (23)$$

The branching ratio in a higher dilepton invariant mass region is also measured. In a higher m_{ll} region the branching ratio suffers from long-distance contributions of J/ψ and ψ' resonances, and a large theoretical uncertainty is expected. However, in $0 < m_{ll}^2 < 6 \text{ GeV}^2$ region, the short-distance contribution dominates, and the branching ratio is sensitive to the new physics contribution. Therefore in this paper, we consider $\mathcal{B}(\overline{B} \rightarrow X_s l^+ l^-)$ in the low dilepton region. The SM prediction of $\mathcal{B}(\overline{B} \rightarrow X_s \mu^+ \mu^-)$ at NNLO of QCD including a QED correction is given by [19]

$$\mathcal{B}(\overline{B} \rightarrow X_s \mu^+ \mu^-)_{1 < m_{ll}^2 < 6 \text{ GeV}^2} = (1.59 \pm 0.11) \times 10^{-6}. \quad (24)$$

The SM prediction is consistent with the current experimental data.

The $\overline{B} \rightarrow X_s l^+ l^-$ decay is described by the effective Hamiltonian

$$H_{\text{eff}}^{b \rightarrow sl^+ l^-} = H_{\text{eff}}^{b \rightarrow s\gamma} - \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* [C_9(\mu_b) O_9(\mu_b) + C_{10}(\mu_b) O_{10}(\mu_b)], \quad (25)$$

where new operators O_9 and O_{10} are defined as

$$O_9 = (\overline{s_L} \gamma_\mu b_L)(\overline{l} \gamma^\mu l), \quad (26)$$

$$O_{10} = (\overline{s_L} \gamma_\mu b_L)(\overline{l} \gamma^\mu \gamma_5 l). \quad (27)$$

In the MGM, the Wilson coefficients C_9 and C_{10} are dominated by the SM contributions, and the SUSY particles can contribute to $\mathcal{B}(\overline{B} \rightarrow X_s l^+ l^-)$ mainly through C_7 . As a result, the SUSY contributions to $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and $\mathcal{B}(\overline{B} \rightarrow X_s \mu^+ \mu^-)$ are strongly correlated.

The branching ratio in the low dilepton mass region is written by [19]

$$\begin{aligned} \mathcal{B}(\overline{B} \rightarrow X_s \mu^+ \mu^-)_{1 < m_{ll}^2 < 6 \text{ GeV}^2} &= (2.19 - 0.543R_{10} + 0.0281R_7 + 0.0153R_7R_{10} \\ &+ 0.0686R_7R_8 - 0.870R_7R_9 - 0.0128R_8 + 0.00195R_8R_{10} \\ &- 0.0993R_8R_9 + 2.84R_9 - 0.107R_9R_{10} + 11.0R_{10}^2 \\ &+ 0.281R_7^2 + 0.00377R_8^2 + 1.53R_9^2) \times 10^{-7}. \end{aligned} \quad (28)$$

Here R_i 's are ratios of the total and the SM contributions,

$$R_{7,8} = \frac{C_{7,8}^{\text{eff}}(\mu_W)}{C_{7,8}^{\text{eff,SM}}(\mu_W)}, \quad R_{9,10} = \frac{C_{9,10}(\mu_W)}{C_{9,10}^{\text{SM}}(\mu_W)}, \quad (29)$$

where C_i^{eff} 's are the effective Wilson coefficients at $\mu_W \simeq m_t$ and they are the LO contributions [19].

$\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ depends mainly on $|C_7|^2$, and it is insensitive to the sign of C_7 . However, $\mathcal{B}(\overline{B} \rightarrow X_s l^+ l^-)$ depends on the sign of C_7 since there is an interference term between R_7 and R_9 . When the sign of C_7 is opposite to the SM prediction, $\mathcal{B}(\overline{B} \rightarrow X_s l^+ l^-)$ becomes too large compared with the current experimental data, and such a possibility is strongly disfavored [20].

3.4 $B_s \rightarrow \mu^+ \mu^-$ Decay

The current upper bound on $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ is given by the CDF collaboration [21],

$$\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) < 1 \times 10^{-7}, \quad (30)$$

at 95% C.L. In the SM $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ is suppressed by the muon mass and quite small. The SM prediction is estimated as $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.35 \pm 0.32) \times 10^{-9}$ [22]. The current experimental upper bound is about two orders of magnitude larger than the SM prediction, and a clue to new physics may be hidden there.

The branching ratio of $B_s \rightarrow \mu^+ \mu^-$ is written by [23]

$$\begin{aligned} \mathcal{B}(B_s \rightarrow \mu^+ \mu^-) &= \frac{G_F^2 \alpha_2^2 \tau_B m_B^5 f_B^2}{64\pi^3} |V_{tb} V_{ts}|^2 \sqrt{1 - \frac{4m_\mu^2}{m_B^2}} \\ &\times \left\{ \left(1 - \frac{4m_\mu^2}{m_B^2} \right) \left| \frac{C_S - C'_S}{m_b + m_s} \right|^2 + \left| \frac{C_P - C'_P}{m_b + m_s} + 2 \frac{m_\mu}{m_B^2} (C_A - C'_A) \right|^2 \right\}. \end{aligned} \quad (31)$$

Here $C_s^{(\prime)}$, $C_P^{(\prime)}$, and $C_A^{(\prime)}$ are the Wilson coefficients which include the SUSY contributions. In Ref. [24], it was pointed out that the amplitude of neutral Higgs boson exchange diagram is proportional to $\tan^3 \beta$ in a large $\tan \beta$ region due to non-holomorphic correction to the Yukawa coupling. The dominant SUSY contribution is written by

$$C_S \simeq -C_P \simeq \frac{m_b m_\mu m_t^2}{4m_W^2 m_A^2} r_b^2 \tan^3 \beta A_t \mu f(m_{\tilde{Q}_L}^2, m_{\tilde{U}_R}^2, \mu^2), \quad (32)$$

and $C'_S = (m_s/m_b)C_S$ and $C'_P = -(m_s/m_b)C_P$. The dominant SUSY contribution to $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$ is proportional to $\tan^6\beta$ and can be enhanced by orders of magnitude when $\tan\beta \gtrsim 10$. Since $r_b < 1$ in the MGM, the $\tan\beta$ enhancement becomes mild. For the numerical calculation we use the complete expression in Refs. [25, 26].

3.5 $B^- \rightarrow \tau^- \bar{\nu}$ Decay

The Belle Collaboration reported evidence of $B^- \rightarrow \tau^- \bar{\nu}$, and the measured branching ratio is [27]

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}) = (1.79_{-0.49}^{+0.56}(\text{stat.})_{-0.51}^{+0.46}(\text{syst.})) \times 10^{-4}. \quad (33)$$

In the SM the branching ratio is given by

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}) = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right) \tau_{B^-} f_B^2 |V_{ub}|^2. \quad (34)$$

Using $|V_{ub}| = (4.31 \pm 0.30) \times 10^{-3}$ [28] and $f_B = 0.216 \pm 0.022$ GeV [29], the SM prediction is $\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}) = (1.6 \pm 0.4) \times 10^{-4}$. The current experimental data is consistent with the SM prediction.

In the MGM, the charged Higgs boson exchange contributes to the $B^- \rightarrow \tau^- \bar{\nu}$ decay. The branching ratio is written by

$$\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}) = \frac{G_F^2 m_B m_\tau^2}{8\pi} \left(1 - \frac{m_\tau^2}{m_B^2}\right) \tau_{B^-} f_B^2 |V_{ub}|^2 \left(1 - r_b \tan^2\beta \frac{m_B^2}{m_{H^-}^2}\right)^2, \quad (35)$$

where the non-holomorphic correction r_b is included [30]. The charged Higgs boson amplitude interferes destructively with SM contribution and become significant when $\tan\beta$ is large. The current experimental data gives the constraint on the charged Higgs contribution. However, a large $\tan\beta$ region is still allowed since the cancellation occurs between the SM and the charged Higgs contributions.

4 Numerical Analysis

In the MGM, there are three input parameters, Λ , M_M , and N . The numerical results do not depend strongly on the difference between M_{M_3} and M_{M_2} , so we assume that $M_{M_3} = 1.3M_{M_2} (\equiv M_M)$ in the following analysis. The gaugino masses are almost determined

by Λ so we will show the numerical result as a function of the gluino mass M_3 . We consider the messenger scale as $M_M/\Lambda = 2, 10, 10^2, 10^3$, and 10^4 . For the quark masses, we use $m_t = 172.5$ GeV and $m_b = 4.7$ GeV [28]. With the inputs at the messenger scale, we can calculate the SUSY breaking parameters at the weak scale by solving the RGE of the MSSM. As discussed in Section 2, $\tan\beta$ and μ are determined from the input parameters by imposing the correct electroweak symmetry breaking. Once all the SUSY breaking parameters at the weak scale are determined, the SUSY contributions to the low-energy observables can be evaluated. For convenience, $\tan\beta$ and μ are shown in Fig. 1 as functions of M_3 for various M_M 's.

First, we discuss the light Higgs boson mass, which is shown in Fig. 2 as a function of M_3 for various M_M 's. We have calculated the Higgs mass using the FeynHiggs code [31]. The Higgs boson mass depends mainly on M_3 (*i.e.* Λ). From the LEP bound on the neutral Higgs boson mass ($m_h > 114.4$ GeV), we can obtain the following lowerbound on the M_3 ,

$$M_3 \gtrsim 800 \sim 900 \text{ (} 800 \sim 1100 \text{) GeV, for } N = 1 \text{ (} 2 \text{).} \quad (36)$$

We find that this gives strong bounds on signatures of the MGM.

It is pointed out in Ref. [6] that the stability of the charge-conserving vacuum gives a lowerbound on M_3 in the MGM. When $\tan\beta$ is quite large, the lightest stau becomes light. In addition, the trilinear coupling of the left- and right-handed staus with the Higgs field, which is enhanced by $\tan\beta$, may destabilize the charge-conserving vacuum. However, it is found from Fig. 1 in Ref. [6] that the lowerbound on M_3 is about $700 \sim 800$ GeV even for $M_M \simeq 2\Lambda$ when $N = 1$, and larger M_M makes the bound weaker. When increasing N , the lowerbound is naively expected to be scaled by \sqrt{N} . Thus, the bound on M_3 from the Higgs boson mass is comparable to or rather stronger than that of the vacuum stability.

Now we discuss the correlation between $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and a_μ for various M_M . It is shown in Fig. 3. The left (right) figure is for $N = 1$ (2). In these figures, the colored shaded regions are the experimental bounds at 1, 2, and 3 σ levels, and the dark shaded region is the SM model prediction with a 1 σ error. The red circle dots correspond to discrete M_3 's with 100 GeV intervals. The allowed regions are plotted with solid lines while the regions experimentally excluded by the light Higgs mass bound are also plotted

with dotted lines.

We can see that the SM predictions for these processes are rather low compared to the experimental data. The sizes of the experimental errors are almost comparable to those for theoretical ones. Now we stand on a stage in which we can pin down a parameter region in models beyond the SM by combining experimental results of low-energy observables.

As discussed in Section 3.1, the SUSY contribution to a_μ are always positive. The magnitude of the SUSY contribution is mainly determined by $M_3(\Lambda)$ and is not so sensitive to M_M and N . On the other hand, the SUSY contributions to $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ depends on all the parameters.

For smaller M_M $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ is larger than the SM prediction, while $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ can be smaller than the SM prediction in relatively small M_3 region. We find that the chargino and the gluino contribution are not significant in small M_M regions, in which the non-SM contributions to C_7 are dominated by the charged Higgs contribution. Since the charged Higgs amplitude always contributes to C_7 constructively, $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ in the MGM becomes larger in small M_M region. For large M_M region, the chargino contribution becomes significant and dominates the SUSY contribution to C_7 . As a result, $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ can be smaller than the SM prediction.

We find that the most favorable region in the MGM from $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and a_μ is $M_M \simeq 2\Lambda$ and $M_3 \simeq 900$ (1200) GeV for $N = 1$ (2). The gluino mass will be covered by the direct SUSY search in the LHC experiment. The light Higgs boson mass is close to the current experimental lowerbound as in Fig. 2.

In Fig. 4, we show the correlation between $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and $\mathcal{B}(\overline{B} \rightarrow X_s \mu^+ \mu^-)$. The SUSY contributions to $\mathcal{B}(\overline{B} \rightarrow X_s \mu^+ \mu^-)$ is negative and rather small. If the constraint from the light Higgs boson mass is taking into account, the deviation is at most few %. Considering both theoretical and experimental uncertainties, it would be very difficult to observe the deviation of $\mathcal{B}(\overline{B} \rightarrow X_s \mu^+ \mu^-)$ in near future.

In Fig. 5, we show the correlation between $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$. $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ becomes large for small M_M regions, since $\tan \beta$ becomes very large ($\simeq 40-50$), and the $\tan^6 \beta$ enhancement on $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ is effective. When we consider the favorable region from $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and a_μ , the branching ratio is estimated as $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-) \simeq (6-7) \times 10^{-9}$. This is still far below the current experimental bound, but with in a reach

of search at LHC.

Finally, in Fig. 6, we show the correlation between $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and $\mathcal{B}(B^- \rightarrow \tau^- \overline{\nu})$. The deviation of $\mathcal{B}(B^- \rightarrow \tau^- \overline{\nu})$ is always negative in the MGM and become significant for small M_M regions. This is because $\tan \beta$ becomes very large in small M_M and the charged Higgs contribution interferes destructively with the SM contribution. At the favorable region from $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and a_μ , $\mathcal{B}(B^- \rightarrow \tau^- \overline{\nu}) \simeq 1 \times 10^{-4}$. Notice that this branching ratio is around the 1σ lower bound of current experimental data. A precious measurement of $\mathcal{B}(B^- \rightarrow \tau^- \overline{\nu})$ at B factories may be helpful for search for signature of the MGM.

5 Summary and Discussion

We have revisited phenomenology in the minimal gauge mediated model, in which $\tan \beta$ is naturally large. We have considered the anomalous magnetic moment of the muon, the branching ratios of $\overline{B} \rightarrow X_s \gamma$, $\overline{B} \rightarrow X_s l^+ l^-$, $B_s \rightarrow \mu^+ \mu^-$ and $B^- \rightarrow \tau^- \overline{\nu}$. When $\tan \beta$ is very large, the SUSY contributions to those observables can be enhanced, and the deviations from the SM predictions are strongly correlated with each other. We have updated the results in [8] taking account of the recent theoretical and experimental developments of $\overline{B} \rightarrow X_s \gamma$ and the anomalous magnetic moment of the muon.

We have shown that the experimental bound on the light Higgs boson gives a strong bound on the MGM and requires relatively heavy SUSY spectrum. We find that the lower bound of the gluino mass is $M_3 \gtrsim 800 \sim 900$ ($800 \sim 1100$) GeV for $N = 1$ (2). This constraint makes the signatures in the MGM less significant.

When the messenger scale is larger than Λ by orders of magnitude, $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ is smaller than the SM prediction, since the chargino loop contribution interferes destructively with the SM one. On the other hand, when the messenger scale is same order of Λ , $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ becomes larger than the SM prediction since the charged Higgs contribution becomes important.

The most recent theoretical calculation of $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ in the SM [9, 10] is 1.4σ lower than the experimental world average [15]. When combining it with a 3.4σ deviation in a_μ , a parameter region in $M_M \sim \Lambda$ and $M_3 \sim 1$ TeV is favored. In the region, $\mathcal{B}(B^- \rightarrow \tau^- \overline{\nu})$

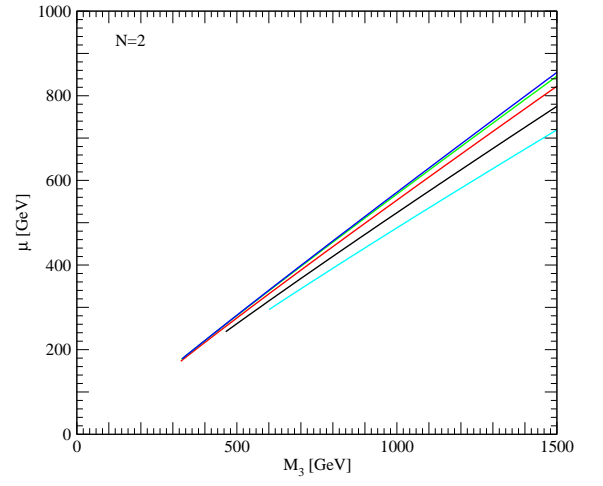
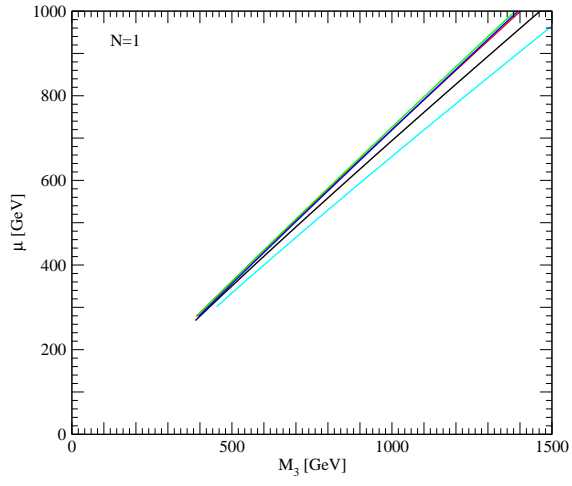
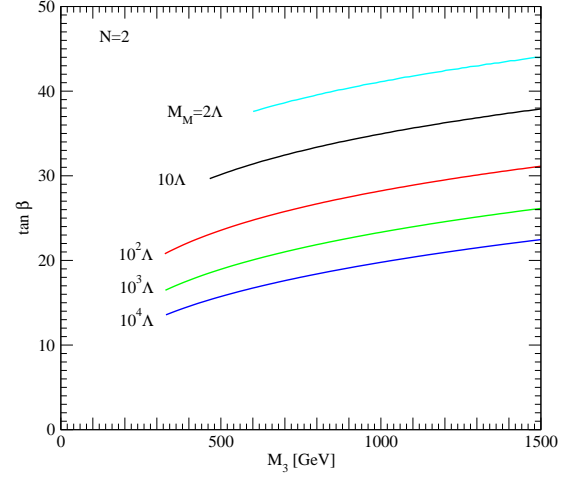
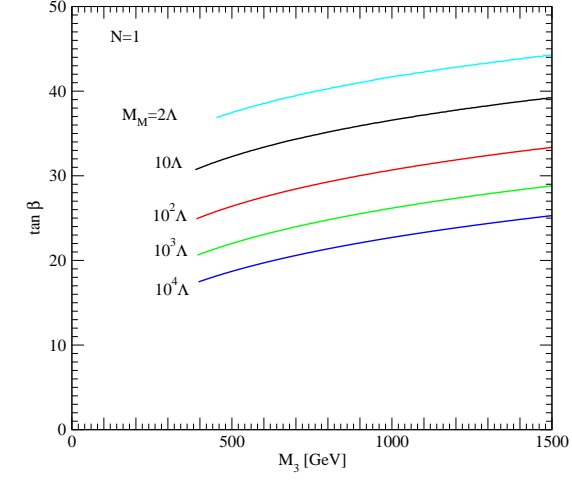


Figure 1: Two parameters, μ and $\tan \beta$, as functions of M_3 for $N = 1$ (left) and 2 (right). Each line in figures of μ parameter is for $M_M = 2, 10, 10^2, 10^3$, and $10^4\Lambda$ from the bottom line.

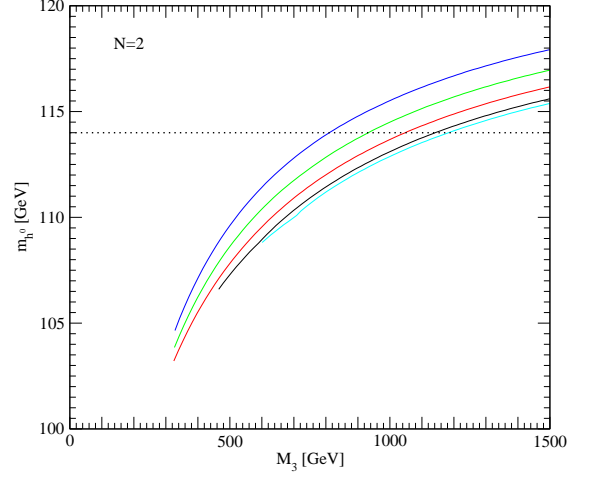
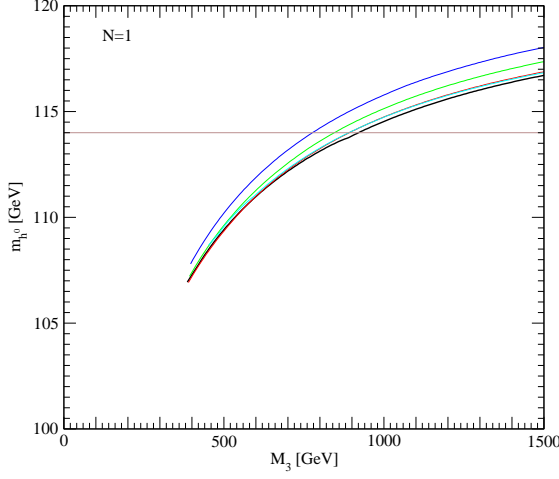
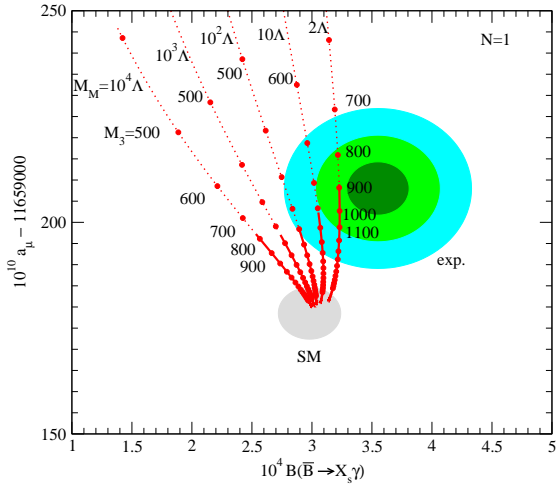


Figure 2: Light neutral Higgs boson mass as a function of M_3 for $N = 1$ (left) and 2 (right). Each line is for $M_M = 2, 10, 10^2, 10^3$, and $10^4 \Lambda$ from the bottom line.



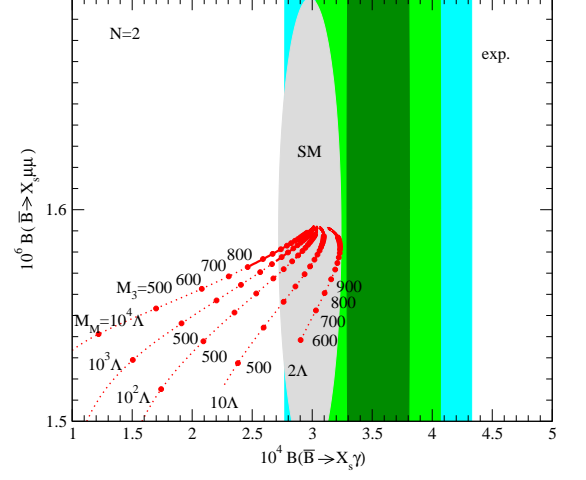
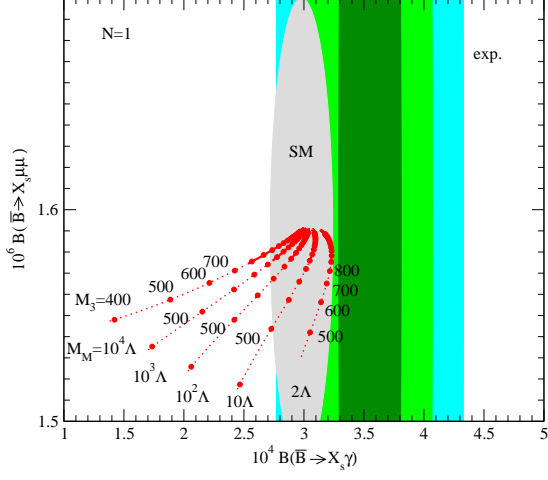


Figure 4: Correlation between $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and $\mathcal{B}(\overline{B} \rightarrow X_s l^+ l^-)$ in the MGM for $N = 1$ and 2.

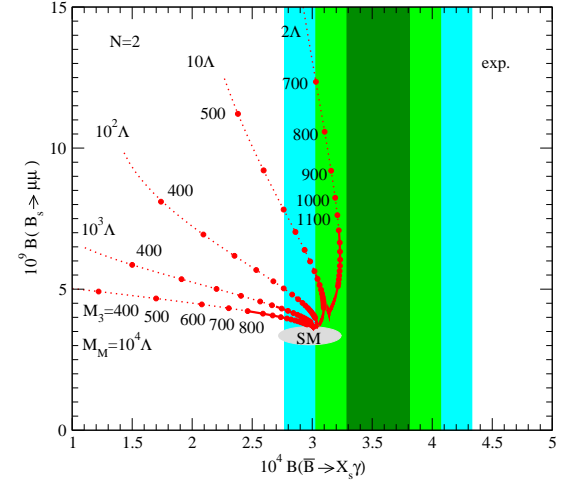
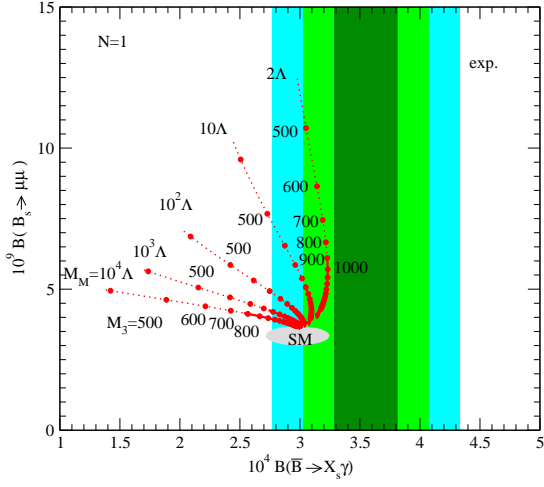


Figure 5: Correlation between $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ in the MGM for $N = 1$ and 2.

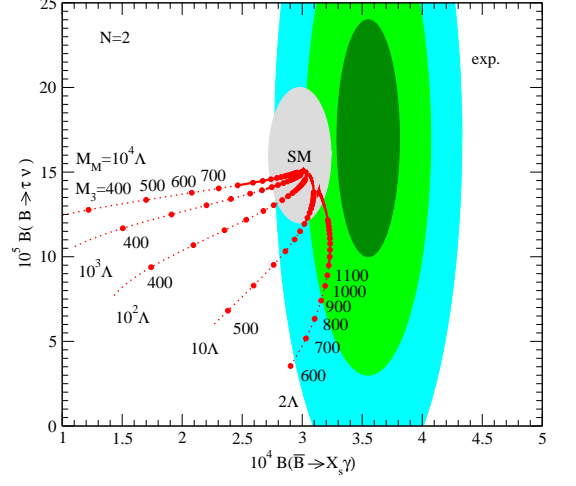
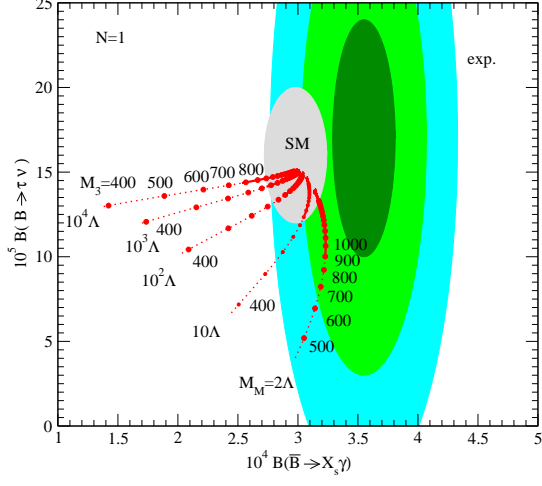


Figure 6: Correlation between $\mathcal{B}(\overline{B} \rightarrow X_s \gamma)$ and $\mathcal{B}(B^- \rightarrow \tau^- \overline{\nu})$ in the MGM for $N = 1$ and 2.

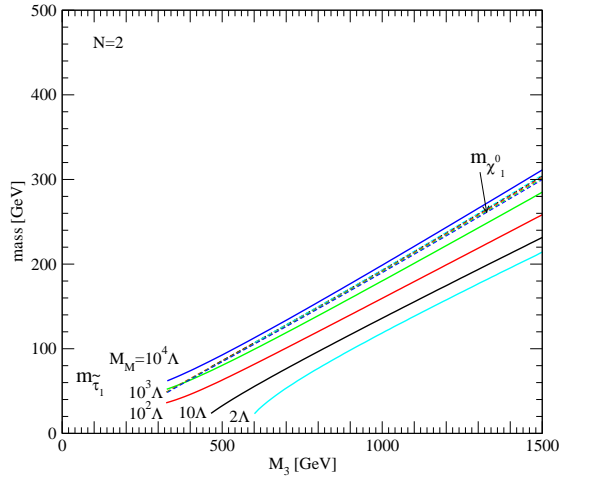
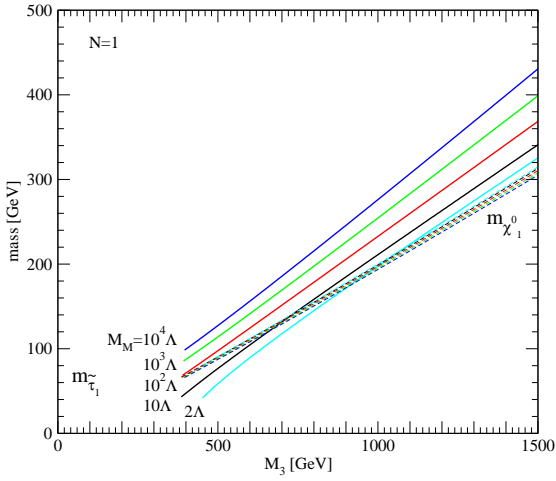


Figure 7: Lightest stau and lightest neutralino masses as functions of M_3 for $N = 1$ (left) and 2 (right).

is predicted smaller than the SM prediction while $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ is larger than the SM prediction. Precise measurements of $\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu})$ at B factories and $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$ at TEVATRON and LHC may play important roles to search for signature of the MGM in addition to SUSY direct search at LHC.

The nature of the next-lightest SUSY particle is important in the gauge-mediated models from viewpoints of collider physics and cosmology. In Fig. 7 we show the lightest stau and lightest neutralino masses as functions of M_3 for $N = 1$ and 2. The stau mass is lighter than or quite degenerate with the neutralino mass in the regions favored from $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ and a_μ . This will have an important implication to SUSY particles searches and also the studies at LHC and LC.

The parameter region $M_M \sim \Lambda$ is well-motivated from the cosmological gravitino problem. When the S field is directly coupled with the dynamical SUSY breaking sector and $M_M \sim 100$ TeV, gravitino may be lighter than ~ 10 eV. Such an ultra-light gravitino is known harmless in cosmology [32].

Acknowledgments

The works of J.H. is supported by the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan (No. 19034001 and No. 18034002). Also that of Y.S. is supported by the BK21 program of Ministry of Education, the KRF Grant KRF-2005-210-C000006 funded by the Korean Government and the Grant No. R02-2005-000-10303-0 from the Basic Research Program of the Korea Science and Engineering Foundation.

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